**T2. Performance and Reliability Analysis of Communication Networks**

Submit to srp@es.aau.dk with subject [NetPerf24][T2][Your Group Name]

**Task 1.** Define entropy H(X) and provide its interpretation.

In information theory entropy is a measure of the uncertainty of a random variable. The entropy is thus a measure for how information, on average, is needed to describe the state of the random variable considering the distribution of the random variable across all the possible states. The mathematical measure can be defined as:

The entropy can be computed in any log base.

In communication theory this is useful for determining the minimum required number of bits to transmit the information contained in each symbol. This gives a binary nature and thus log base 2 is often used in communication theory. The entropy can be used as a goal when designing compression schemes as these should strive to make their average number of bits equal the entropy. This is not possible as entropy is the theoretically best performance, however the goal remains the same, being the close the gap. In communication theory the measure is often in bits per symbol, since the nature of communication theory is digital. In this case a simple example of 1 bit could be thought up and the entropy plotted as a function of probability of success:

Et billede, der indeholder tekst, linje/række, diagram, Kurve

Automatisk genereret beskrivelse

Here we see that the max entropy is found at the point of max uncertainty, in this case at .

**Task 2.** Consider a fair die and *X* a discrete random variable taking values from the set in each role.

1. Calculate its entropy.
2. (ii) Can you generalize the entropy for *M* equally-probable outcomes?

We can calculate the entropy by using the definition:

Since the die is fair, all outcomes are equally probable and we get:

Meaning on average 2.585 bits are needed to transmit the state of X.

We now want to find a form of the entropy when we have M-equally-probable outcomes we use the definition:

Thus we found the general form of M-equally-probable outcomes.

**Task 3.** Derive an expression for following the definition of .

We can expand the entropy to a joint entropy by using the joint probabilities and following the original definition:

Thus we arrive at a new definition for the joint entropy.

It can be visualized using venn diagrams, where the joint entropy is the combination of the entropy in X and in Y:

H(X,Y)

**Task 4.** Show that the entropy of two random variables is the entropy of one plus the conditional entropy of the other:

We have three definitions:

Joint:

Conditional:

Marginal:

We use the definition of Joint entropy together with the chain rule of probability:

We can then part the sum:

We use definition of conditional probability:

The first term then becomes:

Finally we use the marginal distribution by moving the log term out of the sum over y:

And thus, the final term becomes the marginal entropy:

**Task 5.** Show , where is known as

**Mutual Information**. Provide your interpretation of mutual information.

We know that:

eq.1

If and where independent, we would have but in general

eq.2

Thus we get this from eq.1 and eq. 2

The difference between them must be the overlap (mutual information)

Thus like probabilities

Using this we can write:

Similarly for X

Finally, the relationship can be stated: